Gamma Scalping 101 – Gamma/Theta Trading

OptionSellers, LJM, Catalyst are among the prominent fund managers currently facing litigation for large losses due to short gamma positions. Retail investors regularly lose their savings by shorting options as well. It is time to explain a few things about the short gamma and the "gamma scalping" strategies.

This article is split in two parts for convenience:

- Gamma Scalping 101 – Gamma/Theta Trading describes the concept of gamma and theta, the daily P&L of an option market-maker, and the purpose of gamma scalpers. It links options to volatilities and the long-term profitability of the strategy, as well as how gamma-scalpers can select the options to trade.
- Gamma Scalping 102 – The undisclosed risks will explain the not-so-obvious risks associated with the gamma-theta strategy: large and regular losses, the impacts of the gamma distribution and of volatility increases during such large moves, the importance of institutional infrastructure, before concluding with trading recommendations.

What you see on YouTube and probably should not do:

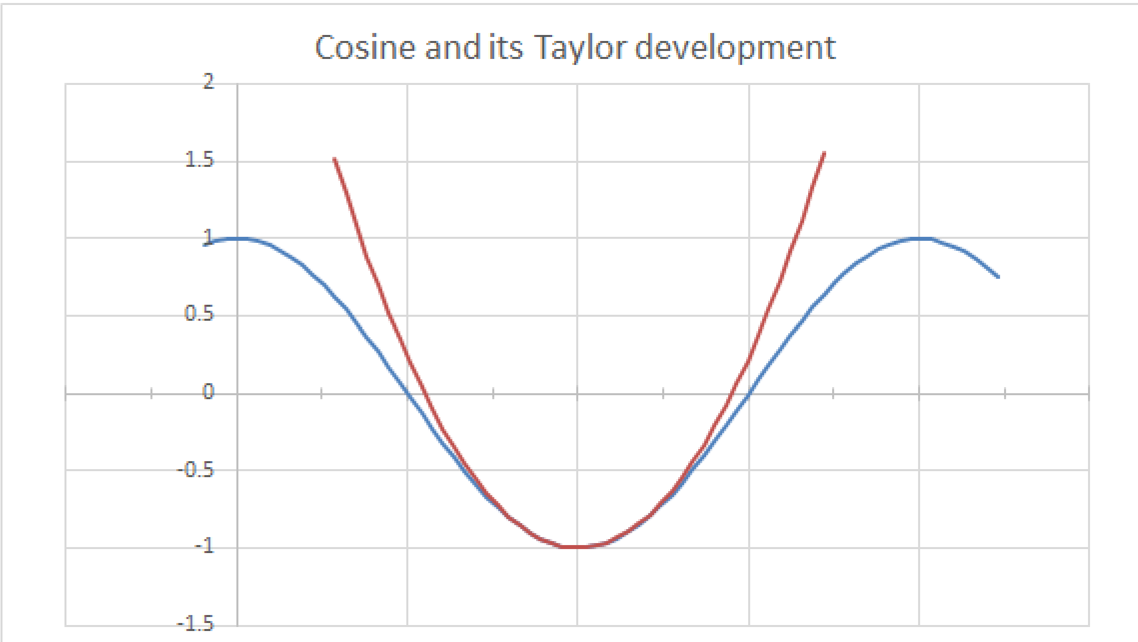
Derivative instruments and their local derivatives

It is difficult to talk about options without a bit of Greek math, but this section hopefully won't be too long.

Many functions of one variable can be locally interpolated with a polynomial. It's called the Taylor development, and it uses the function's derivatives as parameters. As long you don't go too far from the reference point, the polynomial is a good description of the initial function. The formula is:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \cdots$$

Here is the Taylor development for a simple function (cosine) – the polynomial sticks well to the curve.



This is also true for functions of several variables, although expressing the multivariate polynomial quickly becomes cumbersome. For two variables and the first two derivatives (and assuming that partial derivations commute):

$$f(x, y) = f(x_0, y_0) + \partial_x f \cdot dx + \partial_y f \cdot dy + \frac{1}{2}\partial_x^2 f \cdot dx^2 + \partial_x\ \partial_y\ f \cdot dx \cdot dy + \frac{1}{2}\partial_y^2 f \cdot dy^2 \ldots$$

This concept of 'local' description can be applied to many mathematical functions, and in particular to the value of a portfolio of derivatives instruments. That portfolio usually depends on a few well-known variables: s=spot, t=time to maturity, σ=volatility, r=rate, d=dividends.

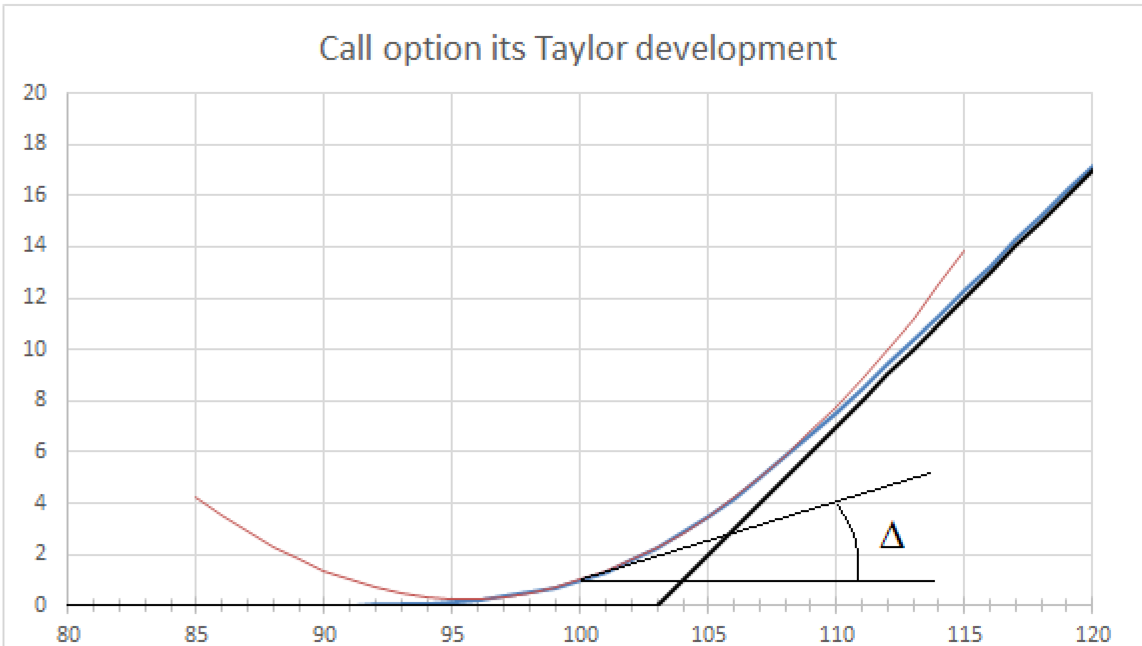
This article will only look at the first derivatives of the book value, except for the spot, where we will consider the first two:

$$P = P_0 + \partial_s P\ dS + \frac{1}{2}\partial_S^2 P\ dS^2 + \partial_t P\ dt + \partial_\sigma P\ d\sigma + \partial_r P\ dr + \partial_{\text{div}} P\ d\text{div} \ldots$$

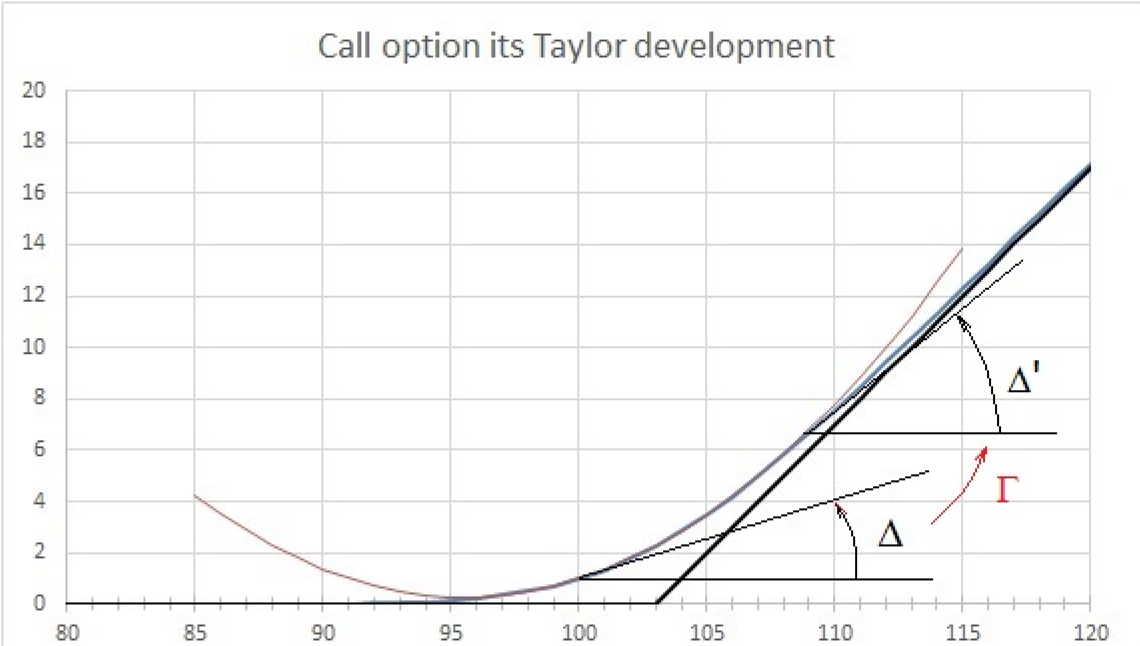
In trading, these derivatives have a name, usually a Greek one:

$$\begin{cases} \Delta = \partial_s P = delta \\ \Gamma = \partial_s^2 P = gamma \\ \theta = \partial_t P = theta \\ \nu = \partial_\sigma P = vega \\ \rho = \partial_r P = rho \\ \epsilon = \partial_{\text{div}} P = epsilon \end{cases}$$

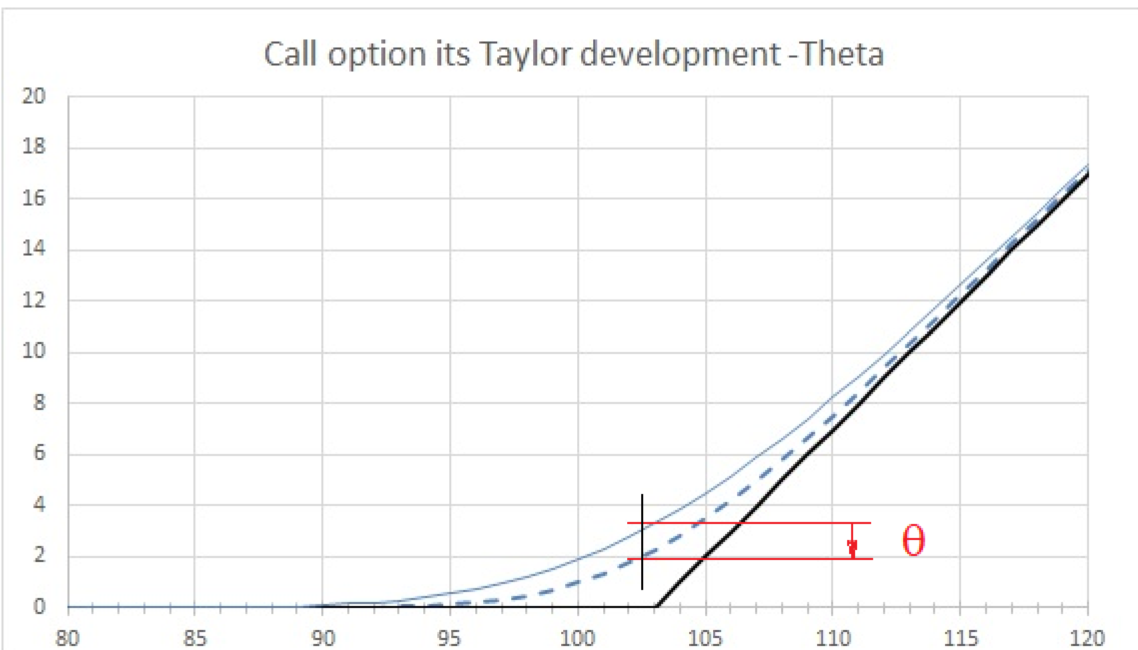
The *delta* expresses by how much your portfolio is equivalent in stock. With a delta of 30%, your portfolio of options behaves like 0.30 share. If the stock gains \$1, your portfolio will gain \$0.30.



The *gamma* expresses by how much this delta is changing when the spot moves. In the example above, if the spot moves by \$1, and your delta is now 35%, then your gamma was 5% per \$. The gamma number allows you to say that, if your stock price moves by $3 instead of $1, then your portfolio will now behave like 30% + 3x5% = 45% of a stock:



Theta is usually called the 'decay' - for every day that passes by, your portfolio's value 'drops' by θ.



The other derivatives are less relevant in this article. *Vega* explains by how much your derivatives increases in value when the implied volatility rises. *Rho* is your interest rate risk and *epsilon* is your dividend risk. There are many others.

Daily P&L of the market-maker, gamma/theta trading

Rates and dividends do not vary much every day and we will ignore them here. Volatility does change daily, but this is not the purpose of this article and we will not consider vega for now.

Since we usually note the theta as a positive value for one day-change, the whole equation simplifies into:

$$P = P_0 + \Delta \cdot \mathrm{dS} + \frac{1}{2}\Gamma \cdot dS^2 - \theta$$

Now, if you are an investor, the delta is the most important aspect of why you trade options. You want the exposure to, say, Johnson & Johnson, as you believe that the stock will go up (call buyer) or down (put buyer).

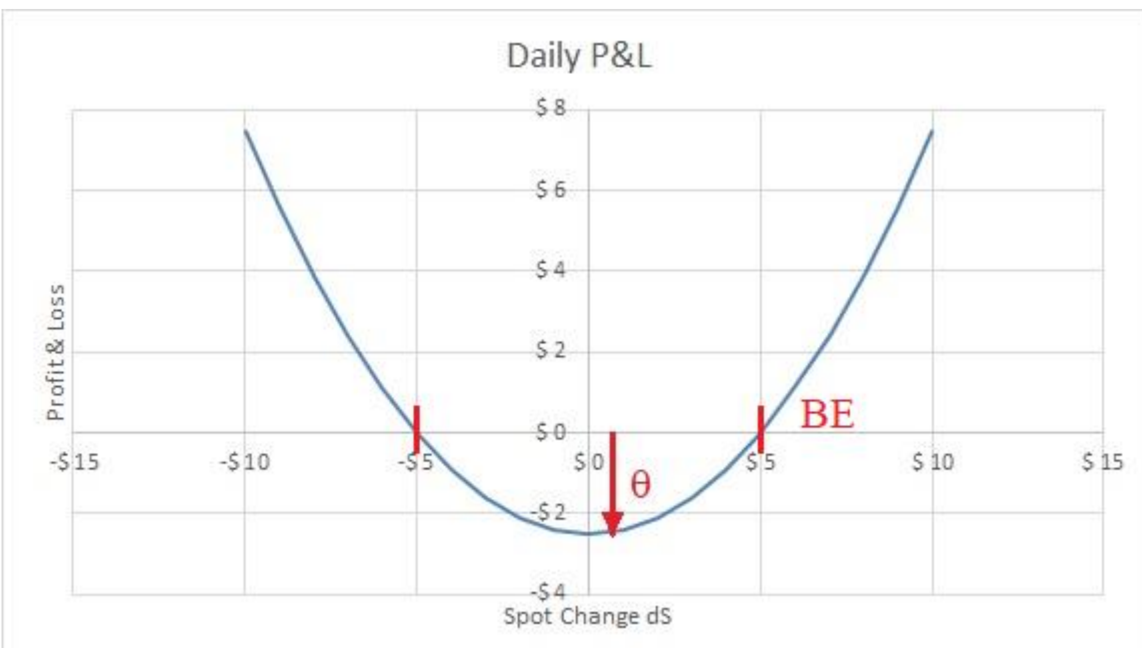
If you are an option market-maker on the other hand, you do not want this delta exposure. Your role is to trade options, hedge all the risks as much as possible, in order to safeguard the small margin attached to the trade. Similarly, a volatility proprietary trader makes gambles on the other parameters (gamma, theta, vega…), but generally not the delta.

As a consequence, the market-maker who just bought a call of 30% delta, will immediately sell 0.30 stocks to 'pass his hedge' or 'hedge his delta' and remove this market exposure.

The market-maker is now left with a simple formula:

$$dP = \frac{1}{2}\Gamma \cdot dS^2 - \theta$$

In other words, on a day-to-day basis, and assuming that the stock doesn't move to much, his P&L at the end of the day will be a parabola:



Interestingly enough, that calculation works in identical fashion with calls, puts, or mix of both; once the delta has been hedged, calls and puts behave virtually in the same way. For market-makers, it is the position of the strike that count, not the type of option or combination of options. A put-spread and a call-spread have the same behavior and risks.

This parabolic P&L shows the “gamma vs theta” effect of options. Once you own an option, you become automatically exposed to the stock when the stock deviates from its previous hedging level. The further the stock moves, the more your exposure increases. As the stock goes up, you become ‘long’ (you own extra shares) *and you collect profits on these extra stocks*. This is a positive benefit, which is the direct consequence of the ‘convexity’ of an option. Without that second derivatives (gamma), this benefit would not exist. The convexity works both on the way up or on the way down – you are not sensitive to the market direction.

But to be entitled to this ‘free profit’ when the stock moves around, you have to own the option, and its value diminishes every day. The price you pay for this benefit is the decay. Your option loses value constantly, and this theta is charged to you daily, no matter what the stock does.

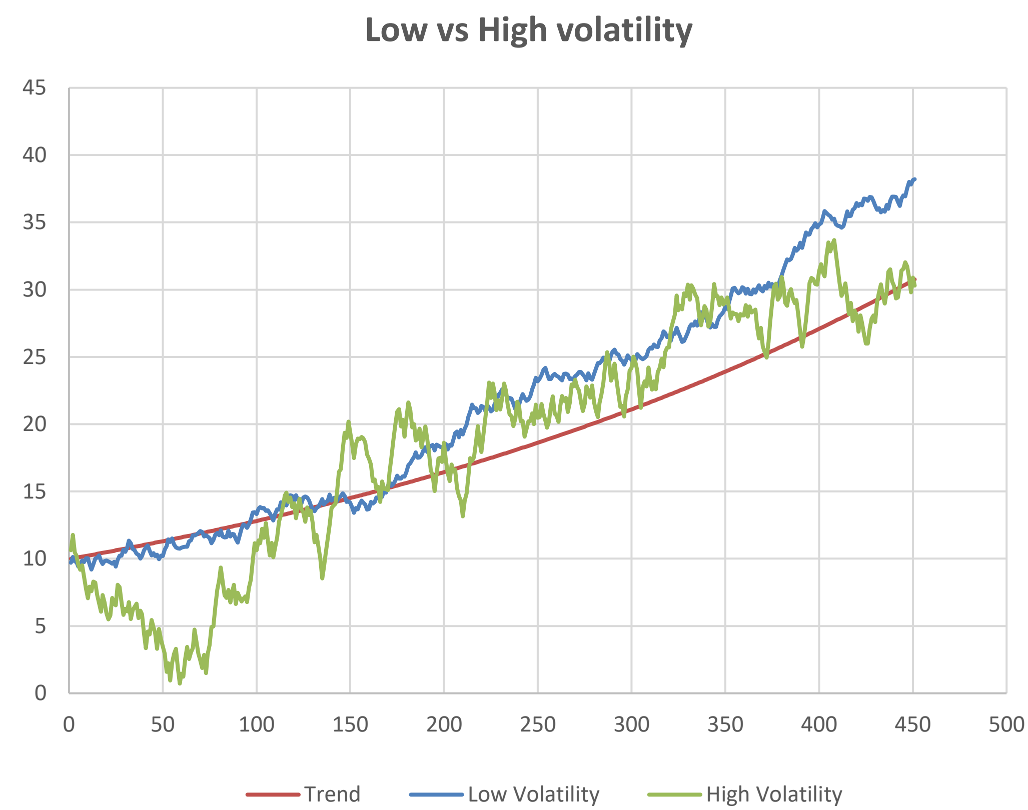
There is a point, a “break-even”, where the effect of the convexity and the theta decay are equivalent. In the graph above, it is at $5. If the stock moves less than $5, the convexity will not be sufficient, and the decay will cost you more than the gamma effect. If the stock moves more than $5, the gamma will ‘pay for the theta’, and you will gain a bit more than your cost. The further the stocks moves afterwards, the more profit you will then have.

Historical vs implicit volatilities, long-term PL

When does the option trader breaks-even on longer time periods? That’s where the historical and implied volatiles come along.

Volatility is the measure of how much a stock moves. A low volatility instrument moves very little (say 0.5% a day on average), while a highly volatile instrument will move 2-3% or more every day. Utility companies and train operators are low-volatility stocks. A biotech company or a drug maker, which can easily move 10% on a news, or a company with a risk of bankruptcy, are high-volatility stocks.

Volatilities indicate what is the ‘typically or ‘standard’ return of the stock every day.



Mathematically speaking, financial volatilities are calculated in a funny way – it is the *annualized* standard deviation of the daily returns. Because we measure volatilities over a period of one year (252 business days), but consider daily moves, there is square root of 252 which appears in the calculations. And since SQRT(252) = 15.9 is close to 16, you have to divide the historical or implied volatilities by 16 to find the standard ‘daily’ move:

Asset type	Typical volatility	Standard daily move
Diversified index (S&P500) = low volatility	12%	12% / 16 = 0.75%
Standard stock	16%	1%
Volatile stock	25-35%	1.6-2.2%
Highly volatile stock	50-80%	3.1-5%

For a typical stock (16% volatility), the average daily move is 1%, in a mix of up and down movements.

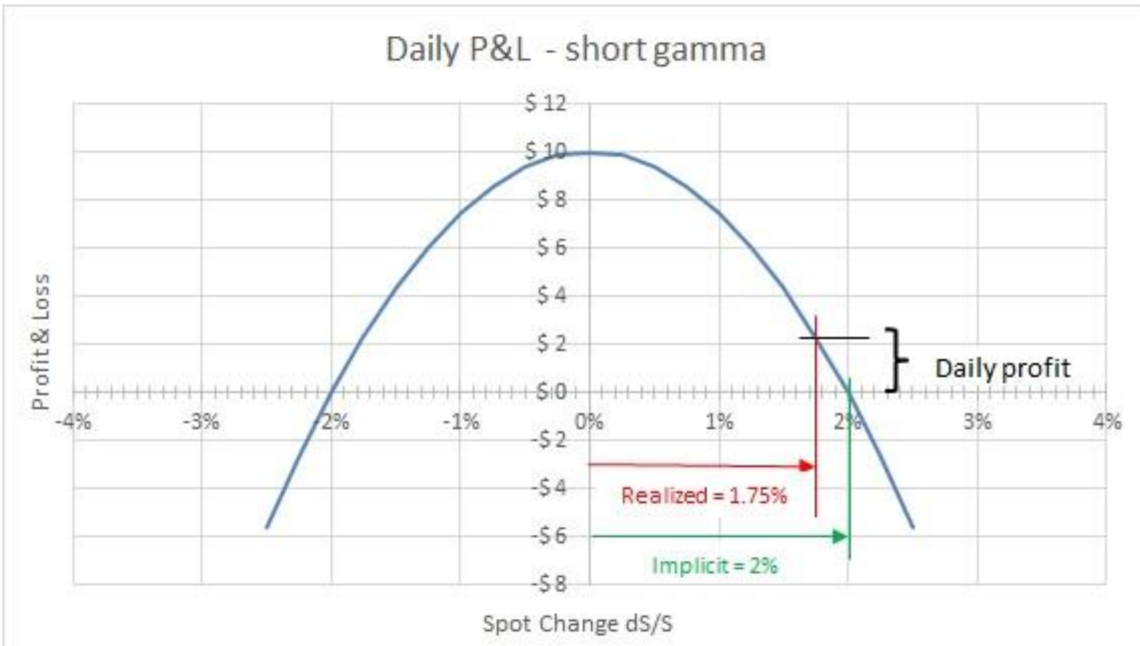
Volatilities varies a lot from an instrument to another, and from a period to another. There are periods of high volatilities (think of the dot-com crash of 2001 or the real-estate crash of 2008), where the entire market moves a lot. There are also quiet periods (like now).

Historical and realized volatilities are not option-related concepts. They simply are a measure of how much a stock fluctuates. ‘Historical’ volatilities at how stocks have moved in the past, while ‘realized volatilities’ indicates how much the stock will be oscillating in the coming weeks or months starting from now.

But since the $ price of options is directly linked to the volatility of the underlying asset, they tend to trade with a volatility input ("implied volatility") similar to where the asset floats.

Perception of the future differs from one moment to another or from one actor to another, and the implicit volatilities are not always in line with the historical volatilities. If market-maker perceive a crash is coming, or when many investors suddenly start buying options (options follow rules of offer and demand), the implicit volatility goes up. There is no guarantee that historical volatilities will follow.

Suppose you sell an option with an implicit 2% daily move (32% implied vol) and that the stock only moves 1.75% per day on average during the lifetime of the option (28% realized), then every day the gamma/theta effect will be a little bit on your favor. The stock, on average, will not reach the break-even of the parabola, and you will keep a bit of money each day:

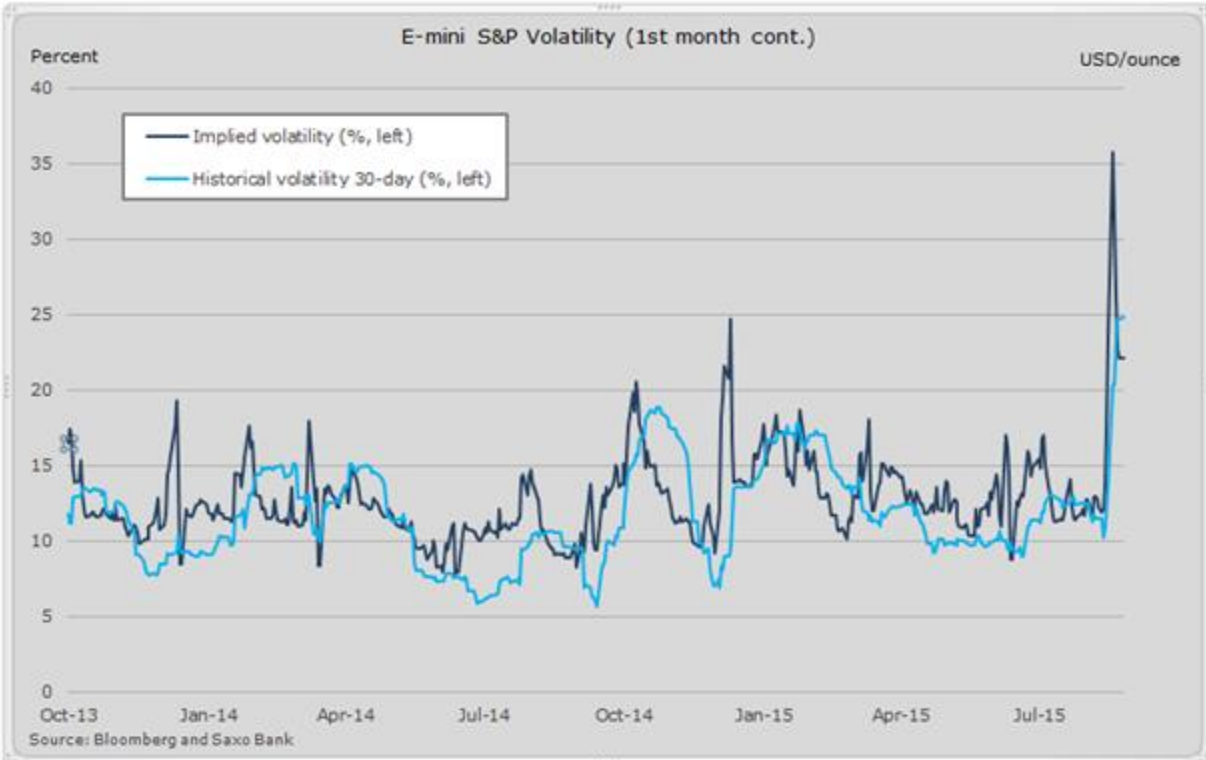


Gamma scalping

Gamma scalpers are the option traders who collect the difference between implied and historical volatilities.

If historical/realized tend to be higher than the implicit volatilities, they have a benefit to buy options and collect more gamma than the theta will cost. On the other hand, if historical/realized tend to be lower than the implicit volatilities, selling options and collecting more theta than the gamma will cost is the way to go.

Looking at the graph below, you will make money by being short gamma/options and long theta when the implied (dark blue) curve is above the historical (light blue):



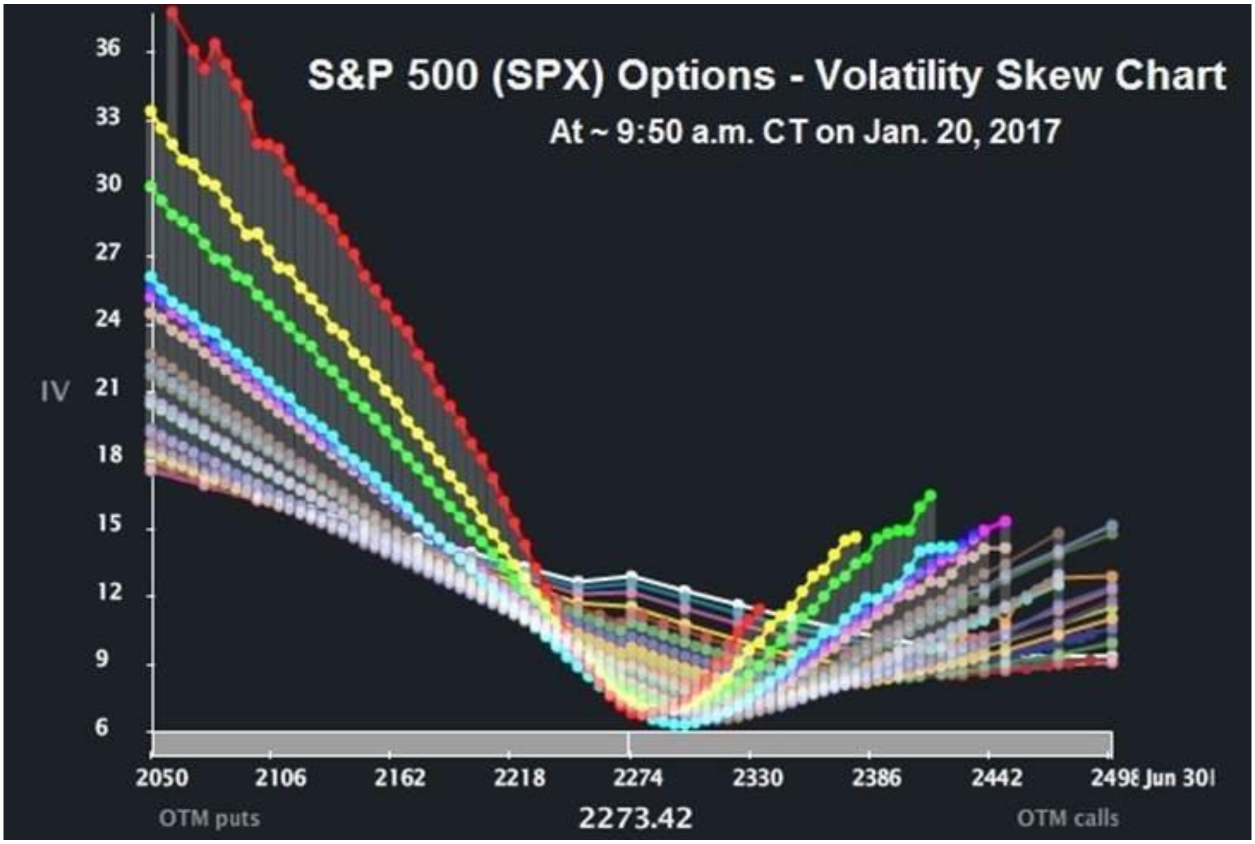
The reality is that, over the last few years, historical volatilities have tended to be stubbornly low, way lower than their long-term averages. Meanwhile, insurance companies, who tend to buy long-dated options for market protection and regulatory reasons, tend to pull the prices of long-dated options up. This propagates down to the front-end to some degree. One against the other, implied volatilities have generally been higher than historical, and it's the short gamma (selling option) which has paid off.

Also, selling options as low as 8-9% on the S&P is really hard to do for most experienced vol traders, as implied volatility tends to mean-revert around a usually higher average level. This trader takes into account the vega effect, which is not explained here.

Selecting the options

Now all options do not have the same implied volatilities, even if they refer to the same underlying asset. Actually, there is a pretty wide range of implicit volatilities at any given time.

Implied volatilities depend on strike position, as well as maturity, creating a surface of volatility, rather than a simple value. The typical volatility surface is like the graph below (lower strikes on the left, higher strikes on the right, each curve representing a different maturity):



Short-dated options (red, yellow and green curves) tend to have a ”V” shape call ‘smile’. Long-dated options (dull colors in the back) tend to have a slope called ‘skew’, where lower strikes are usually more expensive than higher strikes.

The main reasons for this distribution are:

- On the long term, institutional investors buy long-dated puts, lifting up the long lower strikes and all the back of the term structure.
- Other investors sell upper calls in covered call structures or collars (they lose upside gains, but add a yield of option premium to their portfolio). In both cases, these trades cap the medium-to-long upper strikes.
- On the short-term, wing options (small puts and small calls which are way out of the money), are priced in pennies. Paying an extra penny doesn’t cost much to your bank account, for the benefit of a large gain should the stock suddenly go up or down. This is why small options tend to have higher volatilities. Much more on that later.
- As a result, longer-dated options tend to be more expensive than shorter-dated options. The term structure (implied volatilities of ATM option as a function of maturity) tends to be sloping upward.
- This being said, while long-term volatilities are slowly moving, short-term volatilities change much faster, and the short term of the curve frequently has an ‘inverted’ shape.

The consequence of that diversity of implied prices, is that a gamma scalper can select options of higher implied volatility for the same historical volatility, hence maximize the histo/implied difference. There are benefits and drawbacks to that:

- If he sells longer-dated options, he will carry that position for a much longer period. Making the bet between histo and implied becomes much more hazardous.
- Also, longer options have less gamma than shorter options (the options have little convexity), so the gamma/theta play is much harder to put in place. One would need many more options for the same gamma or the same $ amount of theta.
- Meanwhile, the prices of long options are much more sensitive to volatility (1% change in implied volatility represents many $). With long-dated options, you are more betting on where the implied volatility is going, rather than comparing it to the historical. That is a very different kind of strategy.

If you sell short-dated OTM options, you will have a much higher implied level, and still some gamma/theta. Unfortunately, OTM options quickly have no $ premium (trading quickly in pennies rather than dollars), and you will need to sell a lot of options for the same amount of premium. Much more on that later again.

So, the reality is that the trader has to find the right balance between strikes, maturities and vol difference. That's where experience is critical.

Which position to take

Position on the gamma/theta divide (going long gamma or going long theta?) tend to vary.

Market-makers at large broker dealers and hedge funds oscillate between the long and short gamma directions.

- That's first of all because their views on the coming months change regularly: what will happen during Brexit? This company will announce its earnings in two weeks. The world's global macro environment is changing due to the China-US commercial negotiations…
- Sometimes, market-makers have to take trades from clients, and can't always unwind their exposures due to liquidity constraints. If the client gives you a lot of gamma, you have to take it and you might be stuck with it for a long time. Think of a corporation issuing a warrant or a stock-option plan out of the market-maker's book. These are elephant sizes!
- They manage many different assets (say, all the different stocks in continental Europe), and positions will vary from one stock to another.
- Actually, they can play the relative value of one stock against another (Shell tends to be 2 points rich, Exxon is 5 point rich. He will go long gamma on Shell and short on Exxon, with no overall volatility exposure and little market/sector risk).
- Or they can play the structural difference between an index and its components. That's called a dispersion strategy (short gamma on the index and long gamma on the components of the index). That approach is actually a bet on stock correlations.
- Don't forget finally that the book of the trader is under overall risk constraints ("no more than XXX dollars of theta") and so he might have to take a direction just to hedge the overall exposure.

There are also portfolio managers who do not oscillate back-and-forth between the two directions and keep one permanently:

- Insurance buyers will always buy puts. Not only institutional investors and corporations do that, but there are active portfolio managers who sell such exposures to those investors and who will lift the vanilla market for their hedge (think of Nassim Taleb and his black swans…)
- There are individuals and asset managers, like OptionSellers, LJM or Catalyst, who will short options on a continuous basis. They are routine premium collectors. They have done generally well over the last few years, but that is not necessarily the smartest thing to do, as the strategy contains much more risk than meet the eye. More again on that in the second article.

Partial conclusion

This concludes the first of this two-part article. We have seen:

- How the daily P&L of a portfolio of derivatives can be expressed with a simple parabola.
- The concept of break-even, and when gamma (convexity) brings more value than theta (decay).
- How historical and implied volatilities explain the gamma scalper’s long-term P&L.
- How this trader can improve his odds by trading options of higher implied volatility.

The next article will go into more details about

- the risks associated with large stock moves, and how frequent they are.
- How gamma distribution impacts P&L during those large moves.
- The amplifying effect of volatility increase in large moves.
- Why the institutional environment is important for option trading.
- Before concluding with recommendations.

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Gontran de Quillacq, consultant / expert witness

Gontran de Quillacq has over 20 years of experience in portfolio management, derivatives trading, proprietary trading, structured products and investment research. He has worked with top-tier banks and hedge funds in both London and New York.

Background Experience - After reaching PhD and MBA level through six global Ivy leagues, Mr. de Quillacq traded derivatives for two decades, from vanillas to exotics, both proprietary and client-facing, at top-tier banks in the square mile and on Wall Street. As a portfolio manager, he researched and managed investment strategies, delivered both in hedge fund and in structured note formats. He initiated the distribution of investment strategies through derivatives, an activity now called 'portable alpha' and 'smart beta'. For the following five years, Mr. de Quillacq ran due diligence on investments strategies and selected senior investment personnel for some of the world's most famous and most demanding hedge funds and asset managers. In 2017, he co-founded a quantitative activity deploying the latest machine learning techniques in global long/short equities.

Litigation Support - Mr. de Quillacq's own investment experience and his cross-sectional review of other professionals give him unique experience on what can be done, what should be done, what should not be done, and the grey areas in-between. During a personal case, his legal team was so impressed by his wide and thorough knowledge in finance, his capacity to explain complicated ideas in simple terms, and his strong performance on the stand, that they strongly recommended he expand into litigation support services. Mr. de Quillacq is now a FINRA/NFA arbitrator, a member of the Securities Expert Roundtable and an IMS Elite Expert. He has consulting affiliations with **Barrington Financial Consulting Group** and **Ankura** (Navigant).

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